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Redesign of a water table for study of unsteady flow in turbines

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REDESIGN OF A WATER TABLE

FOR STUDY OF UNSTEADY FLOW IN TURBINES

by

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ABSTRACT

The work undertaken in this thesis involved the improvement of existing equipment for the study of unsteady flow in gas and steam turbines through experiments carried out with a water analogue. Some of the phenomena studied were pressure waves generated by the turbine blades as they pass the nozzles, vortices formed by the flow leaving the nozzles, and separation from the suction side of the blades. From this hydraulic analogue, an effort was made to study the phenomena with movie films and to determine the analogous flow rates and turbine speeds at which the unsteady activity occurs.

The improvement of the equipment mainly involved making it sturdier and more adjustable. The water table was placed on a more solid table which could be more easily leveled. It is possible now to control the water flow onto the table more accurately by means of a gravity tank employing an overflow which insures a constant level. This level provides a constant head governing flow to the table. Also, a better method of feeding water to the table was devised. From a constant pressure header around the table, the water is fed onto the table through rubber tubes into filtering boxes, packed with screens. This helps to make the flow onto the table more even and free of disturbances.

Unsteady flow was observed in the form of vortices and waves constantly generated by the blades.

NOMENCLATURE

A	Cross sectional area
a	Speed of sound in a gas
c	Speed of propagation of surface waves
c_p	Specific heat at a constant pressure
c_v	Specific heat at a constant volume
d	Water depth
d_o	Water depth where flow originates at zero velocity
g	Gravitational acceleration
i	enthalpy
i_o	Total enthalpy
M	Mach number
$\dot{m}$	Mass flow rate
P	Pressure
P_o	Pressure at stagnation conditions
Q	Volume flow rate
q_e	Volume flow entering control volume
q_l	Volume flow leaving control volume
R	Universal gas constant
R_g	Radius of gas turbine rotor
T	Temperature
T_o	Stagnation temperature
u	Velocity component in the x direction

V	Velocity
V_g	Gas velocity
V_L	Liquid velocity
V_{max}	Maximum attainable velocity
V_o	Stagnation velocity (zero)
v	Velocity component in the y direction
W_g	Blade velocity- gas turbine
W_L	Blade velocity- water turbine
w	Velocity component in the z direction
x, y, z	Coordinate parameters
x_o, y_o, z_o	Coordinates of point where a streamline originates at zero velocity
γ	Ratio of specific heats
ρ	Density
ρ_o	Density at stagnation conditions
ϕ	Velocity potential

I. PURPOSE AND THEORY

The equipment described in this thesis was built for the study of unsteady flow phenomena in the space between the blades and nozzles of gas and steam turbines. Of particular interest is the effect on the flow of pressure waves generated by the turbine blades. Also, the nature of flow through the blades may be studied at various flow rates and turbine rotor speeds.

The vehicle employed to study these conditions is the well-known hydraulic analogue (3,4). Shallow water with a free surface, under the influence of gravity, flows over a horizontal surface through a radial arrangement of nozzles and blades. Pressure waves in the analogous gas flow will appear as surface waves on the water.

The gas and steam turbines to which the analogue applies are axial, made up of many stages. It is felt, however, that a radial turbine with flow toward the center could just as easily experience the same phenomena. In applying the hydraulic analogue to this problem, a flat table was constructed, the water flow originating from the outer perimeter of the table, flowing radially in through the nozzles which are arranged in a circle, and across the vertical blades which are attached to a horizontal wheel suspended from above. The nozzles consist of curved channels of rectangular cross section.

The hydraulic analogue is valid for flow through a channel of rectangular cross section corresponding to the flow of gas with a specific heat ratio of two. This channel may vary in width, however, at different points in the flow field, and the channel may curve, as long as the flow remains irrotational. In the area of the table between the blades and the nozzles, it may be observed that the analogue is valid if we consider a typical channel to be made up of a nozzle passage and the streamlines leaving the nozzle. Since the water flows radially in toward the center of the table, the width of the effective channel decreases in direct proportion to the distance toward the center of the table.

The flow in the test section (between the nozzles and the blades) may be considered essentially irrotational if the very small vortices formed where the flows from the adjacent nozzles intersect are neglected, as well as the vorticity due to the formation of a boundary layer. In actual operation, disregarding the vortices, the only region of significant vorticity is the boundary layer. If the flow is assumed to be frictionless, there is no boundary layer and the flow may be considered two-dimensional. If the water level is not too low, the boundary layer should not have an overly great effect on the flow. The flow on the water table should exhibit good adherence to the assumptions of the hydraulic analogue for depths approximately equal to one-quarter of an inch.(4).

When it is felt that an unsteady flow phenomenon of interest is being observed with the naked eye, motion pictures are taken. The camera is focused on a small portion of the test section, including the space spanned by two nozzles and the passing blades. The angle which the line of sight makes with the water is adjusted until an overhead fluorescent ceiling light is barely visible on the surface. Then any surface disturbance appears dark against the reflection of the white light. Among the surface activities we seek to observe are vortices, waves generated by the passing blades, and disturbances due to separation of flow.

The speed of the camera is adjustable to sixty-four frames per second. At this speed, many separate frames may be taken, tracing the propagation of surface disturbances as they move across the test section.

It is also desired that the rotor speed and flow rate be determined to record the conditions under which certain unsteady flows exist. The RPM of the rotor is measured by the timing circuit and clock which times the seconds elapsed for one-half of a revolution. This time can easily be translated into revolutions per minute. The flow rate and velocity may be found by employing a relationship derived in the appendix.

$$V = \sqrt{2g(d_0 - d)} \qquad (3)$$

Here, d_0 is measured just upstream of the sluice gate; d is measured at the end of the nozzle passage. V is the fluid velocity leaving the nozzle.

The flow rate is simply the cross sectional area of flow through all of the nozzles multiplied by the above velocity.

$$Q = (\text{no. of nozzles} \times \text{width of nozzle} \times d) \times V$$

The analogous quantities for gas flow can be determined from the relationships in Appendix A.

II. EQUIPMENT

Upon the commencement of this undertaking, equipment already existed and had been used to perform the experiments. Much of this equipment, however, had proven not to be sturdy or accurate enough to obtain the desired results. Consequently, this thesis is concerned with the redesign of those parts and the performance of the experiments.

The apparatus as it existed was made up of a number of components. A plastic table for water flow radially toward the center with nozzles and a wheel with blades fixed to it arranged in a circle constituted the main part. Other parts included a tank under the table to collect the water, a pump to recirculate it, a gravity tank to control the flow rate, an overhead frame to support the blade rotor, and a speed control and timing device.

Of these components, the overhead frame, the blade and nozzle assemblies, and the speed control and timing device were left essentially unchanged, but the other components were redesigned and rebuilt.

A. WATER TABLE

The existing water table was 43 5/8" x 43 5/8" and had six inch high sides. The turbine nozzles were located at the center about the outlet to the collecting tank. In each corner a drain hole was drilled to accommodate a tube to the collecting tank. Around the hole was an overflow box four inches square with sides the height of the desired head behind the sluice gate. For different heads, different height sides were installed on the boxes.

The essential parts of the water table are the rotor wheel and blade assembly and the radially located, two-dimensional nozzles (fig.10). For the purpose of the experiments undertaken, this section must be supplied with water of a constant depth and flowing at a constant velocity, uniform about the table at a given radius. It was decided that the existing circular sluice gate with a constant head of water behind it is the best means of controlling this depth and velocity. However, it was felt that a better method of feeding the water could be devised.

Instead of carrying the water directly to the table surface from the supply tank by many small rubber tubes, as in the original design, it is now piped from the tank through eight 1 1/4 inch diameter clear plastic tubes to a completely enclosed header around the outside of the table. The header (fig. 4, fig.1) was made in four separate sections to accommodate the existing overhead frame (fig.9) which was attached to the support table (fig.6) at the outer limit of the existing plastic table. Each section of the header is supplied with water by two of the plastic hoses. The header was placed around the table in the shape of an octagon so that the distance from the point of discharge onto the table to the sluice gate is approximately equal at all points. The water is fed onto the table from the header by thirty-two small rubber tubes, 1/4 inch in diameter. These tubes, about four inches long, lead to small enclosures of plastic called discharge boxes (fig.7). These boxes, surrounded by screening, help to produce an even depth of water behind the sluice gate and reduce turbulence.

In the existing equipment, the depth of water behind the sluice gate (fig.2) was controlled by overflows which sent the excess water to the collecting tank (fig.4). This was changed so that the rate of incoming water is held constant by holding the level in the supply tank constant by means of an overflow to the collecting tank. Consequently, a constant head in the supply tank produces a constant rate of flow into the header and onto the table; therefore the head behind the sluice gate is constant and controllable.

As the water is fed onto the table through the discharge boxes, there is unavoidably considerable disturbance in the form of extraneous gravity waves and capillarity waves. To combat this problem, three rows of triple thickness wire screens were placed around the entire circumference, just behind the sluice gate. This greatly reduces the surface disturbances propagating into the test area without adding much resistance to flow.

The circular sluice gate is suspended from the overhead frame and its height is adjusted by the threaded rods which fasten it to the frame on four sides. The rods go through holes in the supports on the sluice gate and the nuts which hold the gate steady on the rods are turned to alter the height of the gate.

On the existing table, the water left through a hole in the center of the table, the edge of which is curved downward in a radius of two inches. The flow from the table was smooth except for the fact that when the water was allowed to fall unimpeded from the bottom edge of this nozzle, an unsteady flow resulted on the water table. To remedy this undesirable condition, some sort of filter was needed in the exit nozzle to damp out the intermittent disturbances in the nozzle. Steel wool was tried for this purpose, but it quickly clouded the water with rust. Finally ordinary plastic scouring pads were forced into the nozzle around a tin can with the top and bottom removed to accommodate the rotor shaft. This proved effective in damping the disturbances and also in filtering out any particles in the water which would clog the pump.

In constructing the header, a peculiar problem was encountered. For glueing pieces of plexiglass together, a thin liquid solvent is usually used. This provides a very strong seal if the adjoining surfaces are perfectly smooth. If, however, these surfaces are at all uneven, (as in this case, being cut with a hacksaw) only the high spots join and the solvent evaporates, leaving gaps, resulting in an imperfect seal. To combat this problem, several thick glues including epoxy were tried without success. Finally it was found that by mixing the solvent with fine plexiglass filings a slurry could be made which would fill the cracks and bond strongly to the plexiglass. One is obliged to work very rapidly, however, since this compound dries fast.

B. WATER CIRCULATION SYSTEM (FIG.4)

A means must be available to supply the water table with water at the desired rate and to collect and recirculate it. The supply must be adjustable to any desired rate of flow, and the rate maintained constant for a long period of time. A serious problem discovered in operating the existing circulation system was the formation of rust in the pipes, pump, tanks, etc.. To remedy this situation, it was decided to use plastic, stainless steel, or other corrosion resistant material wherever it was in contact with the water, if possible.

The circulation system consists of the following components: The collecting tank to receive the water as it leaves the table; the pump to provide circulation; a discharge valve to regulate the rate of flow from the pump; the supply tank to provide a constant head, hence a constant flow feeding the table; and the eight 1 1/4 inch plastic tubes leading to the header around the table.

Let us consider the components of the circulation system beginning with the tank used to collect the water as it leaves the table.

1) Collecting Tank

The collecting tank must be corrosion resistant and large enough to accommodate most of the water in the system at any given time, because if the supply of water is shut off, the contents of the water table (fig.1), header, and supply tank (fig.3) would flow into the collecting tank while no water was being removed from it. The necessary volume was computed by computing the volumes of the forementioned components, and considering that, during operation, there must be maintained in the tank an adequate head of water to provide sufficient NPSH for the operation of the pump. To satisfy these requirments, a small plastic garbage container of the proper volume was chosen. A hole was cut near the bottom to attach the pipe leading to the pump.

2) Pump

In order to design the pump, the maximum rate of flow must first be determined. At any point in the flow stream the flow rate is

$$Q = \rho VA \qquad \text{or} \qquad \frac{Q}{\rho} = VA$$

As previously shown, in the nozzle passages

$$V = \sqrt{2g(d_0 - d)} \tag{3}$$

From the design of the nozzles, it is known that the total cross sectional area of the nozzle passages is

$$A = (17.1)d$$

The conditions of maximum flow are assumed to be one-half inch in the nozzle passages and three-quarters of an inch behind the sluice gate, i. e., d = 1/2", d = 3/4". Consequently

$$V = \sqrt{2g(d_0 - d)} = \sqrt{2(32.2)(.75 - .50)(1/12)}$$

$$V = 1.16 \text{ ft/sec}$$

$$A = (17.1)d = (17.1)(.50)(1/144)$$

$$A = .0594 \text{ ft}^2$$

therefore

$$\frac{Q}{\rho} = VA = (1.16 \text{ ft/sec})(.0594 \text{ ft}^2)$$

$$\frac{Q}{\rho} = .0688 \text{ ft}^3\text{/sec}$$

For the purpose of sizing a pump, flow rate is better expressed in gallons per minute.

$$\frac{Q}{\rho} = .0688 \frac{\text{ft}}{\text{sec}} \times 60 \frac{\text{sec}}{\text{min}} \times 62.5 \frac{\text{lb}}{\text{ft}^3} \times \frac{1}{8.66} \frac{\text{gal}}{\text{lb}}$$

$$\frac{Q}{\rho} = 29.8 \text{ GPM}$$

The other important factor in sizing a pump is the total head. To pump into the supply tank, the pump must overcome a vertical rise of approximately eight feet and a pressure drop due to friction of no more than two feet of water. Several pumps were examined and their characteristic curves compared. Finally a Meyers one-half horsepower centrifugal pump was chosen. Its capacity is fifty GPM at the total head against which it pumps in this system. This provides an acceptable safety factor, since the maximum flow rate required is about thirty GPM. At the usual operating rate of about fifteen GPM, the pump's efficiency is near its maximum of sixty-five per cent.

Some pumps with corrosion-proof casings, such as stainless steel, phenolic, bronze, etc., were considered, but, they were found to be much too expensive. The pump selected has a bronze impeller and a cast iron casing. The interior of the casing was painted after purchase with corrosion-protective paint to prevent rust from the casing from discoloring the water.

3) Discharge Valve

In order to throttle the flow into the supply tank and thus insurethat there was sufficient flow to fill the tank, but not too much for the overflow to handle, a valve was installed in the discharge pipe. The best type of valve for this purpose is a gate valve because it provides finer adjust ment than any other type. With a thought to corrosion-resistance, a one inch brass gate valve was chosen for the purpose.

4) Supply Tank (fig.3)

The tank which existed previously for feeding water to the table by gravity was approximately 15 inches by 12 inches by 7 1/2 inches deep. It carried water to the table by means of twenty-eight 1/4 inch rubber tubes which originated from the bottom of the supply tank and were fastened to the vertical sides of the water table and discharged directly onto the table. This arrangement was rather awkward. It was fraught with such problems as excessive pressure loss in the tubes, differing pressure losses from tube to tube, and leakage. The rate of flow could be controlled only by the discharge valve downstream of the pump. Because of this, it was impossible to adjust the valve accurately enough to prevent the level in the tank from rising or falling in a short period of time. Of course, when this level changed, so did the rate of flow onto the water table, and consequently, the depth and velocity in the nozzle passages also changed. The overflow boxes on the table did not hold the level behind the sluice gate constant. It was felt that this component of the apparatus could be greatly improved upon.

A new tank was designed which performs basically the same function and is called the supply tank because it accepts water at a rate other than the desired rate and delivers it at an adjustable, constant rate. The water enters the supply tank at one end through a plastic tube and filter. The end of the tube discharges the water near the bottom of the tank behind a fixed, five inch high partition which separates this section from the rest of the tank. Water fills this section and flows over into the main part of the tank. When the main section is filled, the water which does not leave through the tubes at the bottom flows over another partition, this one three inches high, into the overflow section and is carried to the collecting tank by a plastic tube. Obviously the level in the tank is always constant, regardless of the flow rate into the tank. Thus a constant head forces water to the water table at a constant flow rate. This head may be varied by raising or lowering the tank.

The vertical adjustment of the supply tank is accomplished by virtue of the way it is supported. A table with a plywood top and metal frame was constructed to the same height as the water table. The top of this table is 23 inches by 44 inches. The supply tank is supported above this table by means of three 5/8 inch threaded rods, each of which is fastened to the table through a hole by two nuts. The bottom of the supply tank which extends out past the sides, has three corresponding holes through the threaded rods pass. The tank is supported by a nut on each rod. Three rods instead of four were used so that the tank might be easily leveled. A very fine adjustment of the flow rate is provided by raising or lowering these nuts.

5) Inlet Tubes to Water Table

Eight of these tubes carry the water from the supply tank to the header on the water table. An important consideration in piping the water between these two points is the pressure loss. The rate of flow entering the table from each quarter of the header must be the same. Therefore, with a constant head in the supply tank, a greater pressure drop between the tank and the table would reduce the flow in a given tube because the flow rate is related to the pressure drop(fig.4). Tygon clear plastic 1 1/4 inch tubing was chosen because of its smoothness which permits a flow with low resistance. Also, each of the eight tubes were cut exactly the same length, regardless of the distance from the supply tank to the header. This was done because the pressure drop increases with the length of the tube.

6) Piping

The pipe is one inch plastic which is corrosion-proof and easier to assemble than metal piping because the elbows and other connections are cemented together. Metal piping must be screwed or soldered into place.

C. MAIN SUPPORT TABLE (FIG.6)

To support the water table itself, in lieu of the existing table which was not sturdy enough, a table was required which would be heavy and sturdy, isolated from vibrations from the floor, and possessing a precise leveling adjustment to insure a uniform depth of water over the entire table. The top of this table is a hard maple workbench 1 1/4 inch thick, 44 inches by 44 inches. A nine inch diameter was cut through its center to accommodate the exit nozzle of the water table. The table legs were each made from two two by four pieces of lumber thirty inches long, fastened together by means of bolts, and fastened to the table by means of angle brackets. The leveling of the table was accomplished by using three legs instead of four, as in the original design, and making the length of these legs variable. To vary the length of the legs, horizontal two by four blocks were attached to the bottom of the legs and a 5/8 inch threaded rod was placed through a hole in the block and held by a nut and washer on either side of the block. Thus height is adjusted by moving the nut under the block either up or down and tightening the upper nut to hold the position. To enhance the stability of the table, diagonal support braces mutually join the three legs. These supporting members were made of one inch slotted steel angles.

For the purpose of vibration isolation, two rubber vibration absorbing pads were placed between the bottom of each leg and the floor. Two pads achieved isolation without interfering with the stability of the table, i. e., making it wobbly.

D. SUPPLY TANK TABLE (FIG.8)

The table which supports the supply tank is also used as a surface on which to place the controls of the apparatus. Among these are the switch which turns on the pump, the control box which regulates the speed of the turbine rotor, the clock for timing the rotor speed, and the timing circuit with its solenoid switches and battery.

The table has a plywood top 23 inches by 44 inches by 3/4 inch. This top is mounted on a thirty-six inch high frame constructed from slotted steel angles which may be cut to the desired length and bolted together with ready made braces.

The supply tank table also mounts a brace which steadies the vertical pipe from the pump to the supply tank.

E. MOVIE CAMERA

The movie camera used to film the unsteady activity was a Bolex sixteen millimeter reflex camera with speeds up to sixty-four frames per second. The lens aperture used in filming was f/2.8.

III. RESULTS AND CONCLUSIONS

It was discovered that unsteady activity shows on the surface of the flow most clearly when the depth is from .25 inches to .35 inches. It is suspected that at depths of less than .25 inches, the effects of the boundary layer damp and distort the gravity waves. At depths greater than .50 inches, vertical accelerations become significant and the analogy fails.

One of the most difficult problems in the construction of the equipment was vibration isolation. The water table had to be isolated from the floor, the pump and the electric motor. The slightest vibration transmitted to the table causes extraneous surface disturbances on the water.

A drawback to applying the hydraulic analogue to this design was found in the nozzle passages. The surface tension causing the water to adhere to the sides of the nozzles, forming a miniscus, causes a distortion of any wave phenomenon in the vicinity of the nozzle sides. Also, particles of dirt stuck to the nozzle sides cause standing waves to form in the nozzle passages. It was necessary to wet the sides of the nozzles before observing the flow. A detergent was added to the water to reduce the surface tension, but it was only partially effective. Sudsing prevented more than a small amount of this agent from being added.

It was also found that the disturbances generated by the rotating blades propagated back through the nozzle passages, reflected from the sluice gate and traveled back to the test area, causing undesirable extraneous disturbances. This condition was corrected by placing a screen between the sluice gate and the nozzles. These disturbances were effectively damped out.

Waves generated by the blades were seen on the water surface propagating toward the nozzles, as well as vortices shedding from the edges of the nozzles, and vortices caused by separation of flow on the suction side of the blades.

APPENDIX A :

MATHEMATICAL DEVELOPMENT OF THE HYDRAULIC ANALOGUE (3,4)

The hydraulic analogue relates the two-dimensional, irrotational, isentropic flow of an ideal gas with the two-dimensional, irrotational flow of an incompressible, perfect fluid over a horizontal surface under the influence of gravity.

Since it is often difficult and expensive to record data of gas flow through turbomachinery or around objects in a free stream, the analogue provides an excellent tool for inexpensively obtaining qualitative, and in many cases, quantitative information concerning the behavior of gas flow.

In the case of the flow of the liquid, data is easy to record. In flows of one-quarter to one-half of an inch deep, low water velocities correspond to velocities of gases in the sonic range. For this reason, many of the phenomena occurring in the flow may be observed with the naked eye and recorded with less sophisticated devices than would be necessary in the case of a high speed gas. The surface gravity waves can be observed visually.

The basis of the analogue is the similarity in the form of the equations of motion of the gas and of the liquid under consideration.

In describing the liquid flow, we refer to figure 12. The flow originates, at zero velocity, from a large reservior of constant depth, d_0. First we will consider an arbitrary streamline beginning at $x_0 = 0$, y_0, z_0. Of interest are the properties at x_0, y_0, z_0 and at x, y, z. Through a steady, irrotational, incompressible flow field

$$\frac{V^2}{2} + \frac{P}{\rho} + gz = \text{constant} \quad \text{(Bernoulli's equation)}$$

or

$$P + \frac{\rho V^2}{2} + \rho gz = Po + \frac{\rho V_0^2}{2} + \rho g z_0$$

This relationship may also be expressed as

$$V^2 = 2g(z_0 - z) + \frac{2(P_0 - P)}{\rho} \qquad (1)$$

Assuming vertical acceleration is negligible compared to gravitational acceleration, and that atmospheric pressure is chosen as the reference point (gauge pressure), pressure in the liquid is proportional to the distance below the surface (d. - z). Therefore

$$P_0 = \rho g(d_0 - z_0)$$

$$P = \rho g(d - z) \qquad (2)$$

Substituting these relationships into (1)

$$V^2 = 2g(z_0 - z) - \frac{2\cancel{\rho}g(d_0 - z_0 - d + z)}{\cancel{\rho}}$$

$$V^2 = 2g(\cancel{z_0} - \cancel{z} + d_0 - \cancel{z_0} - d + \cancel{z})$$

$$V^2 = 2g(d_0 - d) \qquad (3)$$

Since this relationship is independent of z_0 and z, it is seen that the velocity at a given x and y distance from the origin is the same at any depth. It is known that for a gas

$$V^2 = 2g(i_o - i) \tag{4}$$

Consequently, the depth of the liquid is analogous to the enthalpy of the gas.

Also, for the liquid

$$V_{max}^2 = 2gd_o \tag{5}$$

$$\left(\frac{V}{V_{max}}\right)^2 = \frac{2g(d_o - d)}{2gd_o} = \frac{d_o - d}{d_o} \tag{6}$$

For the gas

$$V_{max}^2 = 2gi_o$$

$$\left(\frac{V}{V_{max}}\right)^2 = \frac{i_o - i}{i_o} = \frac{c_p(T_o - T)}{c_pT_o} = \frac{T_o - T}{T_o} \tag{7}$$

Therefore it can be seen from (6) and (7) that

$$\frac{d_o - d}{d_o} \text{ corresponds to } \frac{T_o - T}{T_o}$$

and

$$\frac{d}{d_o} \text{ corresponds to } \frac{T}{T_o} \tag{8}$$

Therefore, in determining the velocity ratio $\frac{V}{V_{max}}$, $\frac{d}{d_o}$ is analogous to $\frac{T}{T_o}$.

Next, we employ the continuity equations of both the gas and the liquid flow considered, in order to relate the gas density to the liquid depth. A result of this derivation is the establishment of a value of the ratio of specific heats $\gamma = c_p/c_v$ for which the analogue is valid.

For the two-dimensional flow of the gas, the continuity equation is

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \tag{9}$$

with this in mind, we examine a differential element of the liquid. See figure 13. As previously stated, the velocity V is the same at any depth, so the velocity is the same at any part of this element of finite height.

For a liquid with a free surface in a gravitational field with negligible vertical acceleration (fig. 12).

$$P = \rho g(d - z) \tag{2}$$

therefore

$$\frac{\partial P}{\partial x} = \rho g \frac{\partial d}{\partial x} \quad \text{and} \quad \frac{\partial P}{\partial y} = \rho g \frac{\partial d}{\partial y} \tag{10}$$

Consider the element of liquid in figure 13 as a volume fixed in space. The continuity equation for this volume is

mass in = mass out

For an incompressible fluid, ρ is constant so

volume in = volume out

$$dq_e = dq_1$$

$$dq_e = uddy + vddx$$

$$dq_1 = \left(u + \frac{\partial u}{\partial x}dx\right)\left(d + \frac{\partial d}{\partial x}dx\right)dy + \left(v + \frac{\partial v}{\partial y}dy\right)\left(d + \frac{\partial d}{\partial y}dy\right)dx$$

$$dq_e - dq_1 = 0$$

$$uddy + vddx - \left[\left(u + \frac{\partial u}{\partial x}dx\right)\left(d + \frac{\partial d}{\partial x}dx\right)dy + \left(v + \frac{\partial v}{\partial y}dy\right)\left(d + \frac{\partial d}{\partial y}dy\right)dx\right] = 0$$

$$\cancel{uddy} + \cancel{vddx} - \cancel{uddy} - u\frac{\partial d}{\partial x}dxdy - d\frac{\partial u}{\partial x}dxdy - \overset{0}{\cancel{\frac{\partial u}{\partial x}\frac{\partial d}{\partial x}dxdxdy}}$$
$$- \cancel{vddx} - v\frac{\partial d}{\partial y}dydx - d\frac{\partial v}{\partial y}dydx - \overset{0}{\cancel{\frac{\partial v}{\partial y}\frac{\partial d}{\partial y}dydydx}} = 0$$

$$- u\frac{\partial d}{\partial x}dxdy - d\frac{\partial u}{\partial x}dxdy - v\frac{\partial d}{\partial y}dydx - d\frac{\partial v}{\partial y}dydx = 0$$

$$u\frac{\partial d}{\partial x} + d\frac{\partial u}{\partial x} + v\frac{\partial d}{\partial y} + d\frac{\partial v}{\partial y} = 0$$

$$\frac{\partial(du)}{\partial x} + \frac{\partial(dv)}{\partial y} = 0 \qquad (11)$$

This is the continuity equation of a steady, incompressible fluid flow with a free surface. When compared to the continuity equation for two-dimensional gas flow

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \qquad (9)$$

Since both equations (9) and (11) have the same form, the density, ρ, of the gas is analogous to the depth of the liquid, d. In both cases ρ or d varies from point to point in the x,y plane. For the gas, ρ, varies due to compressibility; for the liquid, d varies due to gravity. Thus from the continuity equation

ρ is analogous to d

so

$$\frac{\rho}{\rho_0} \text{ is analogous to } \frac{d}{d_0} \tag{12}$$

From Bernoulli's equation

$$\frac{T}{T_0} \text{ is analogous to } \frac{d}{d_0} \tag{8}$$

But, for an adiabatic process of an ideal gas

$$\frac{\rho}{\rho_0} = \left(\frac{T}{T_0}\right)^{\frac{1}{\gamma - 1}} \tag{13}$$

The equations (8), (12), and (13) are satisfied simultaneously only if $\gamma = 2$.

Consequently, the hydraulic analogue is valid for gases having a specific heat ratio in the vicinity of two or in observing phenomena which are not strongly influenced by its value.

The pressure ratio $\frac{P}{P_0}$ also has an analogous hydraulic quantity. For an ideal gas

$$\frac{P}{P_0} = \frac{\rho}{\rho_0}\frac{T}{T_0}$$

$$\frac{\rho}{\rho_0} \text{ is analogous to } \frac{d}{d_0} \quad \text{when } \gamma = 2$$

$$\frac{P}{P_0} \text{ is analogous to } \frac{d}{d_0}\,\frac{d}{d_0}$$

$$\frac{P}{P_0} \text{ is analogous to } \left(\frac{d}{d_0}\right)^2$$

Perhaps the most important of the analogous variables of the analogue is the identity between the speed of sound in a gas and the velocity of waves on the surface of the liquid. In most of the applications of the hydraulic analogue, it is unsteady flow and the formation of mach and shock waves that is of interest. The analogy will be developed using Bernoulli's equation, the continuity equation, and the predetermined condition of irrotationality.

Beginning with Bernoulli's equation, we previously altered its form to equation (3)

$$V^2 = 2g(d_0 - d) \tag{3}$$

solving for d

$$d = d_0 - \frac{V^2}{2g}$$

Referring to figure 13

$$V^2 = u^2 + v^2 + w^2$$

Because u and v are constant throughout the depth of the fluid at a given (x,y) point, and w must be zero at the bottom surface (to maintain contact with that surface), w may be neglected in comparison with u and v. Consequently

$$V^2 = u^2 + v^2$$

$$d = d_0 - \frac{(u^2 + v^2)}{2g}$$

$$\frac{\partial d}{\partial x} = -\frac{1}{2g}\left(\frac{\partial(u^2)}{\partial x} + \frac{\partial(v^2)}{\partial x}\right)$$

$$\frac{\partial d}{\partial x} = -\frac{1}{2g}\left(2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x}\right)$$

$$\frac{\partial d}{\partial x} = -\frac{1}{g}\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x}\right) \qquad (14)$$

Also-

$$\frac{\partial d}{\partial y} = -\frac{1}{g}\left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y}\right) \qquad (15)$$

Now, introducing the continuity equation

$$\frac{\partial(du)}{\partial x} + \frac{\partial(dv)}{\partial y} = 0 \qquad (11)$$

which may be expressed as

$$d\frac{\partial u}{\partial x} + u\frac{\partial d}{\partial x} + d\frac{\partial v}{\partial y} + v\frac{\partial d}{\partial y} = 0 \qquad (16)$$

Substituting (14) and (15) into (16)

$$d\frac{\partial u}{\partial x} + u\left[-\frac{1}{g}\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x}\right)\right] + d\frac{\partial v}{\partial y} + v\left[-\frac{1}{g}\left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y}\right)\right] = 0$$

$$d\frac{\partial u}{\partial x} - \frac{u^2}{g}\frac{\partial u}{\partial x} - \frac{uv\partial v}{g\partial x} + d\frac{\partial v}{\partial y} - \frac{vu\partial u}{g\partial y} - \frac{v^2}{g}\frac{\partial v}{\partial y} = 0$$

Dividing by d

$$\frac{\partial u}{\partial x} - \frac{u^2}{dg}\frac{\partial u}{\partial x} - \frac{uv\partial v}{dg\partial x} + \frac{\partial v}{\partial y} - \frac{vu\partial u}{dg\partial y} - \frac{v^2}{dg}\frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x}\left(1 - \frac{u^2}{dg}\right) + \frac{\partial v}{\partial y}\left(1 - \frac{v^2}{dg}\right) - \frac{uv}{dg}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = 0 \tag{17}$$

If the flow of the liquid is irrotational,

$$\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

In an irrotational, simply connected flow field, a velocity potential $\phi(x,y)$ exists such that

$$u = \frac{\partial \phi}{\partial x} \quad \text{and} \quad v = \frac{\partial \phi}{\partial y}$$

Expressing u and v in terms of $\phi(x,y)$ in (17) in abreviated derivative notation

$$\phi_{xx}\left(1 - \frac{\phi_x^2}{dg}\right) + \phi_{yy}\left(1 - \frac{\phi_y^2}{dg}\right) - 2\phi_{xy}\frac{\phi_x\phi_y}{dg} = 0 \tag{18}$$

From gas dynamics, it is known that the differential equation of the velocity potential for a two-dimensional, compressible flow is

$$\phi_{xx}\left(1 - \frac{\phi_x^2}{a^2}\right) + \phi_{yy}\left(1 - \frac{\phi_y^2}{a^2}\right) - 2\phi_{xy}\frac{\phi_x\phi_y}{a^2} = 0 \tag{19}$$

Since the form of (18) and (19) is the same, it is obvious that a^2 is analogous to dg.

$$a = \sqrt{dg} \tag{20}$$

From hydraulics it is known that the velocity of surface waves in shallow water is $\sqrt{dg}$. So the speed of sound in a gas corresponds to the velocity of a gravity wave on shallow water.

The same relationship may be arrived at through the analogy between water depth and gas enthalpy.

$$a^2 = g\gamma RT \qquad \text{(ideal gas)}$$

$$a^2 = g \frac{c_p}{c_v} RT$$

$$a^2 = g \frac{R}{c_v} c_p T$$

$$a^2 = \frac{g(c_p - c_v)i}{c_v}$$

$$a^2 = g(\gamma - 1)i$$

When $\gamma = 2$

$$a^2 = gi$$

$$a = \sqrt{gi} \qquad \text{and} \qquad c = \sqrt{gd}$$

Since d is analogous to i,

c is analogous to a (20)

The Mach number is determined from the flow velocity and speed of sound.

$$M = \frac{V}{c} = \frac{\sqrt{2g(do - d)}}{\sqrt{gd}}$$

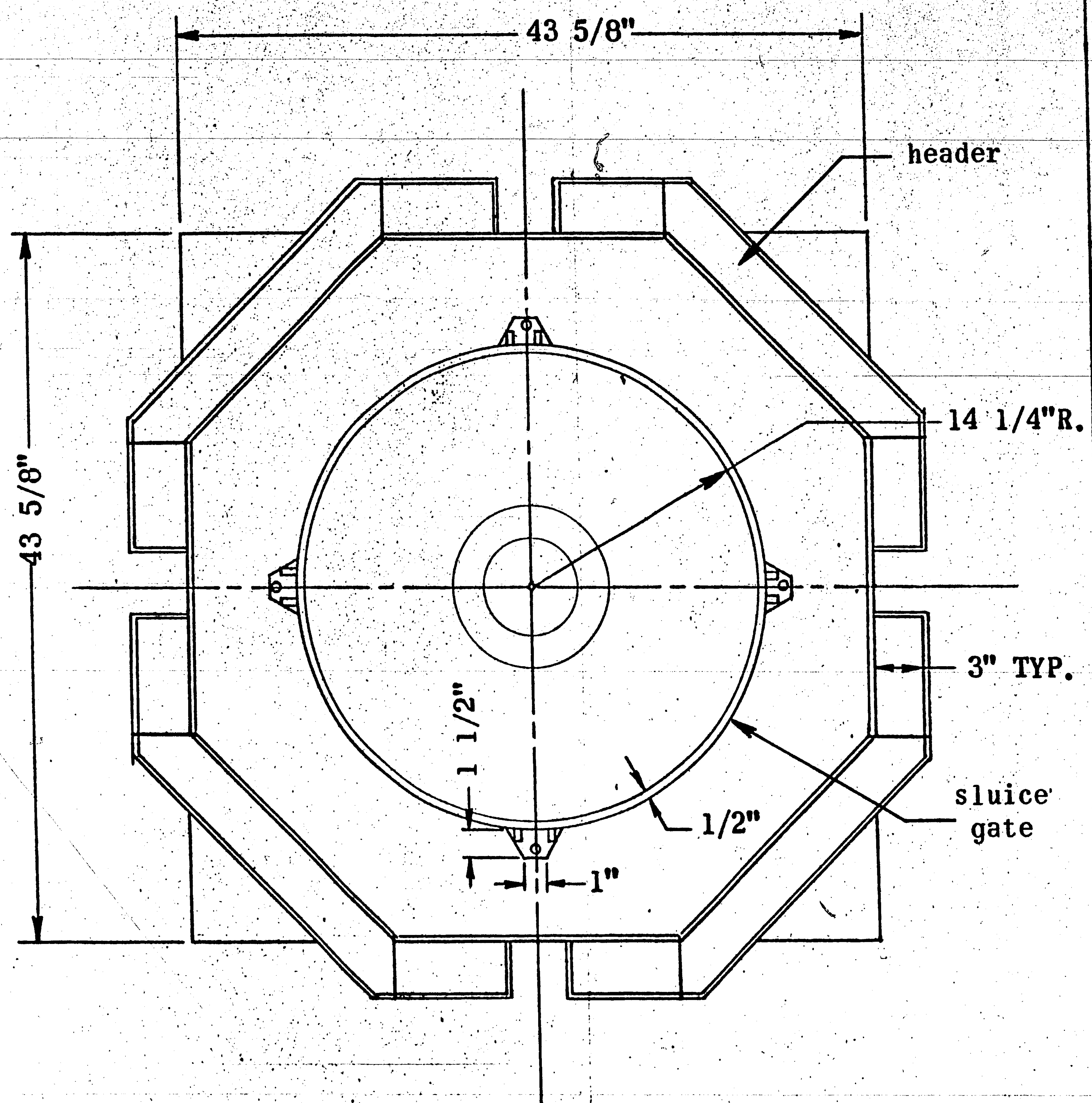
$$M = \sqrt{\frac{2(d_0 - d)}{d}} \qquad (21)$$

Summary:

Two-dimensional, irrotational, isentropic flow of an ideal gas with $\gamma = 2.0$	Two-dimensional, irrotational flow of an incompressible, perfect fluid with a free surface
Speed of pressure waves, $a = \sqrt{\gamma RT}$	Speed of gravity waves, $\sqrt{gd}$
Mach number, $M = V/a$	Froude number, $V/\sqrt{gd}$
Temperature ratio, T/T_o	Depth ratio, d/d_o
Density ratio, ρ/ρ_o	Depth ratio, d/d_o
Pressure ratio, P/P_o	Square of depth ratio, $(d/d_o)^2$

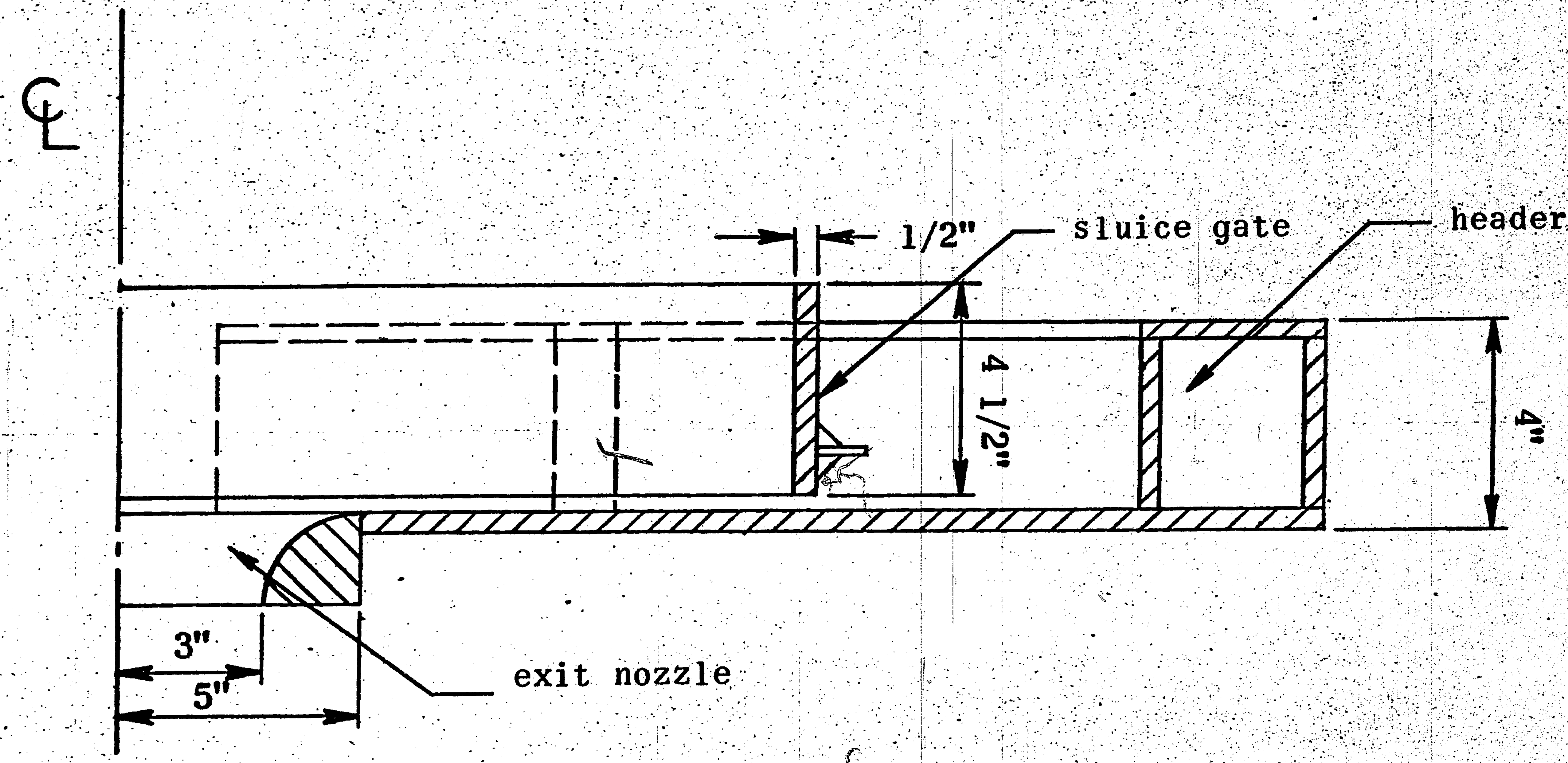
APPENDIX B

Drawings and sketches



WATER TABLE
TOP VIEW

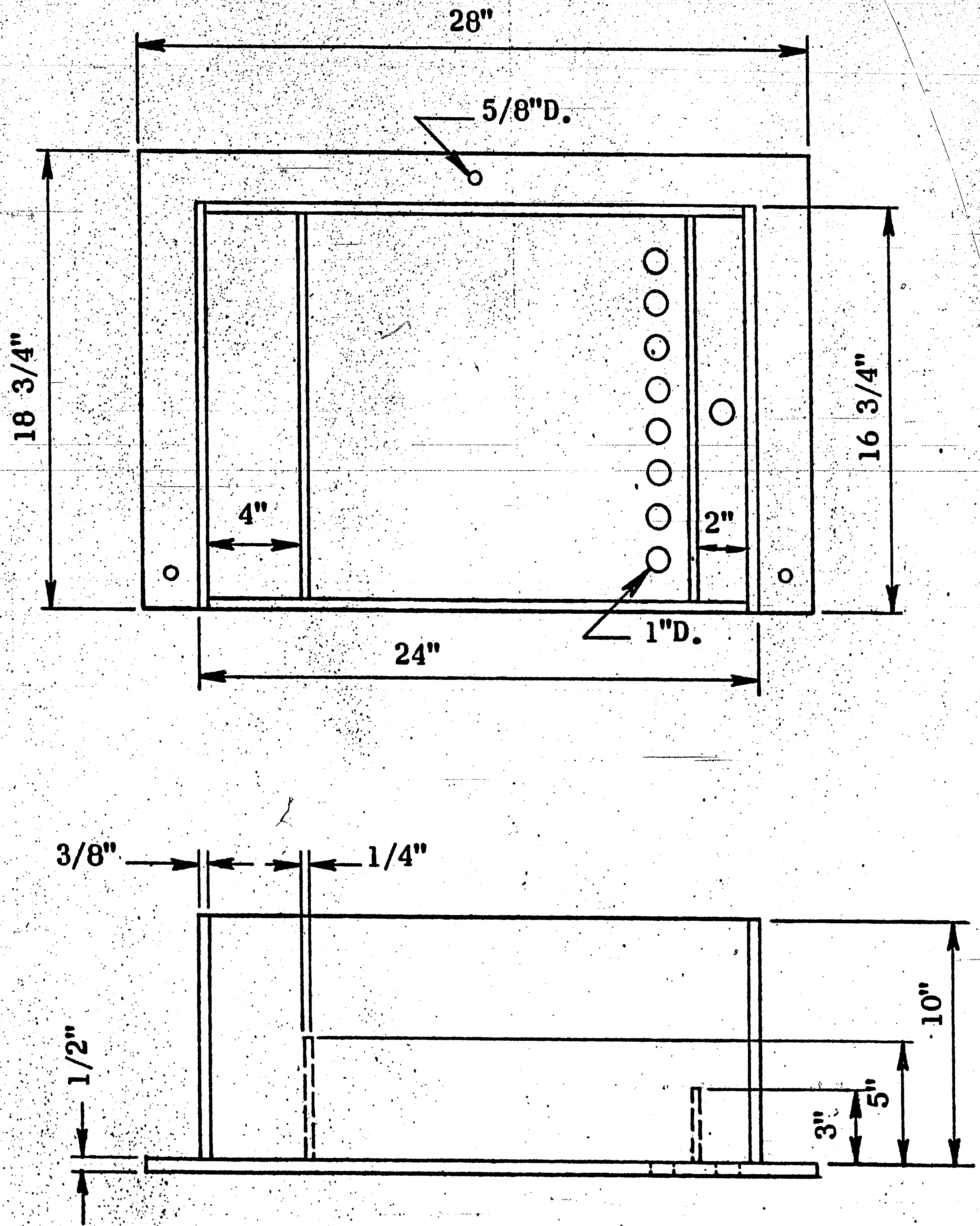
Fig. 1



WATER TABLE

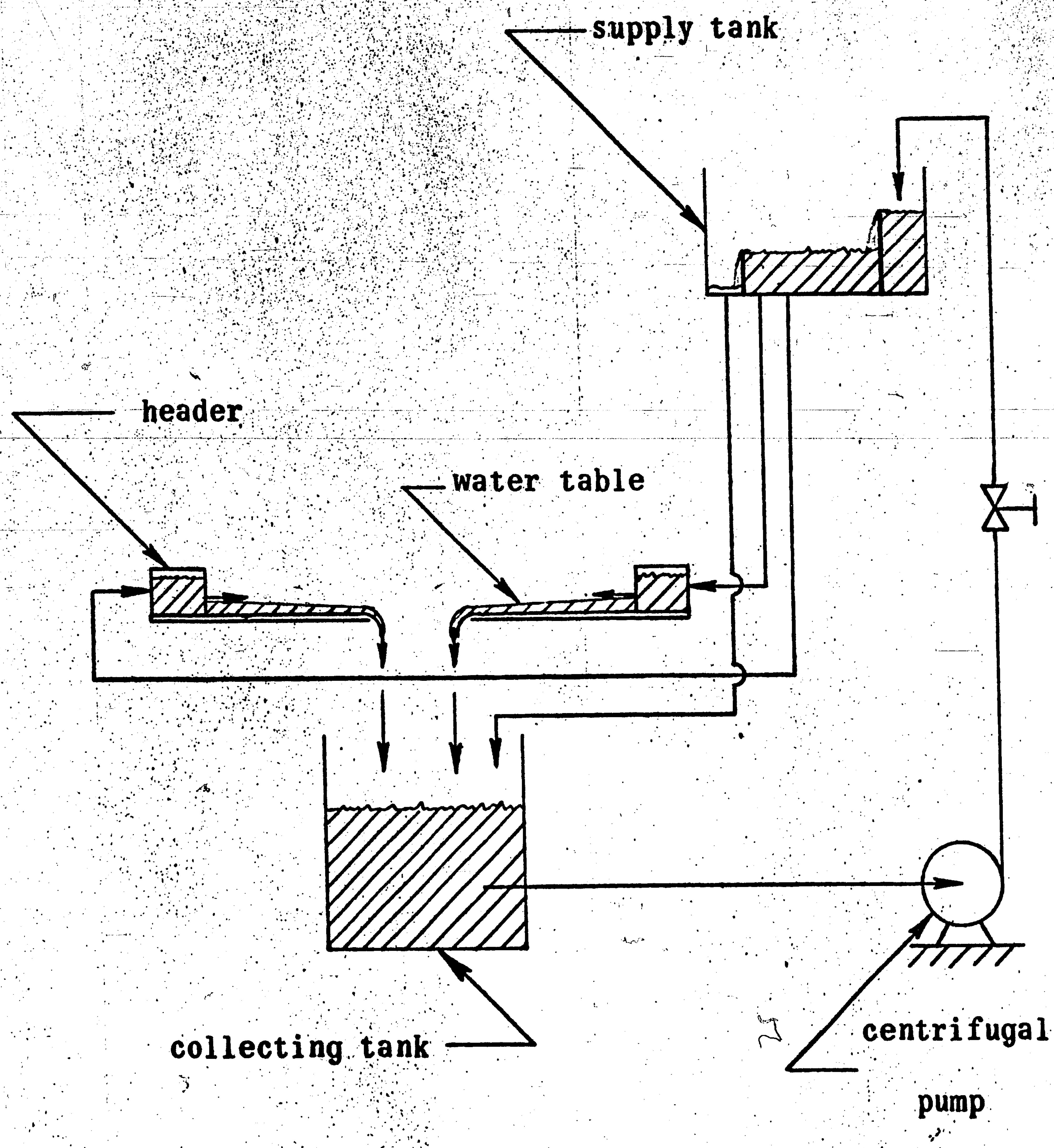
CUT-AWAY SIDE VIEW

Fig. 2



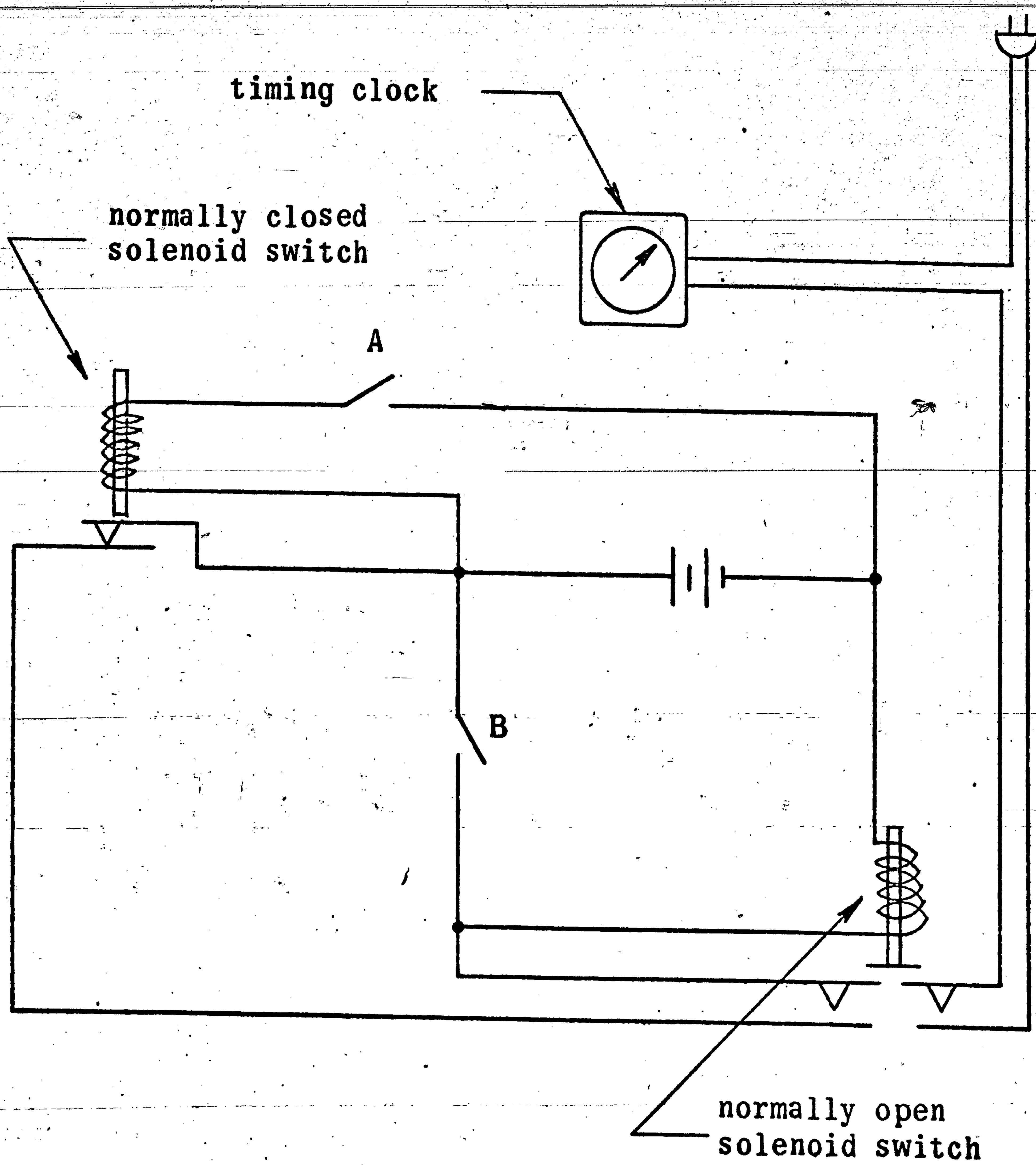
SUPPLY TANK

Fig. 3



CIRCULATION SYSTEM

Fig. 4

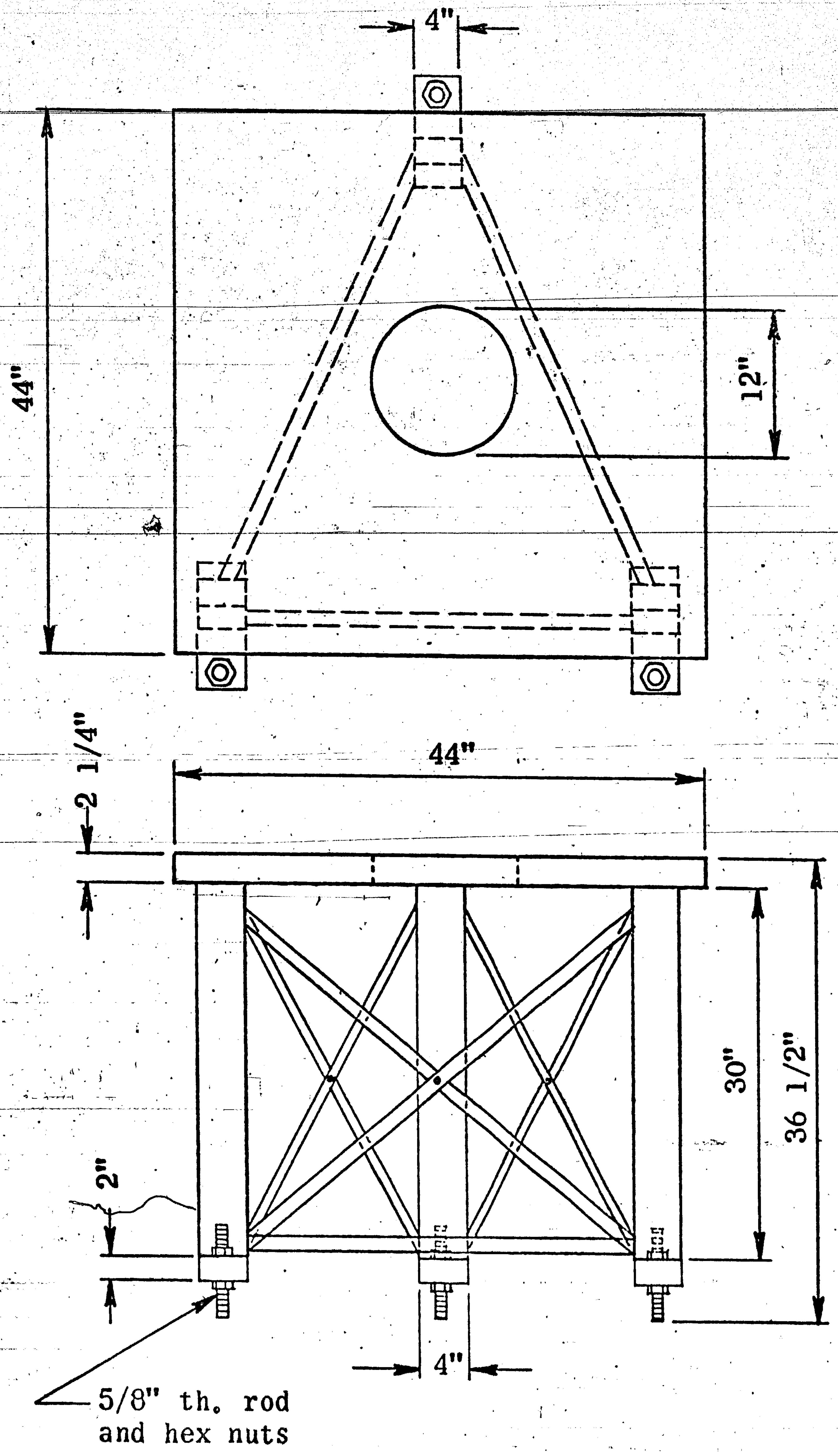


Note:

Contacts A and B are tripped momentarily by the turbine rotor.

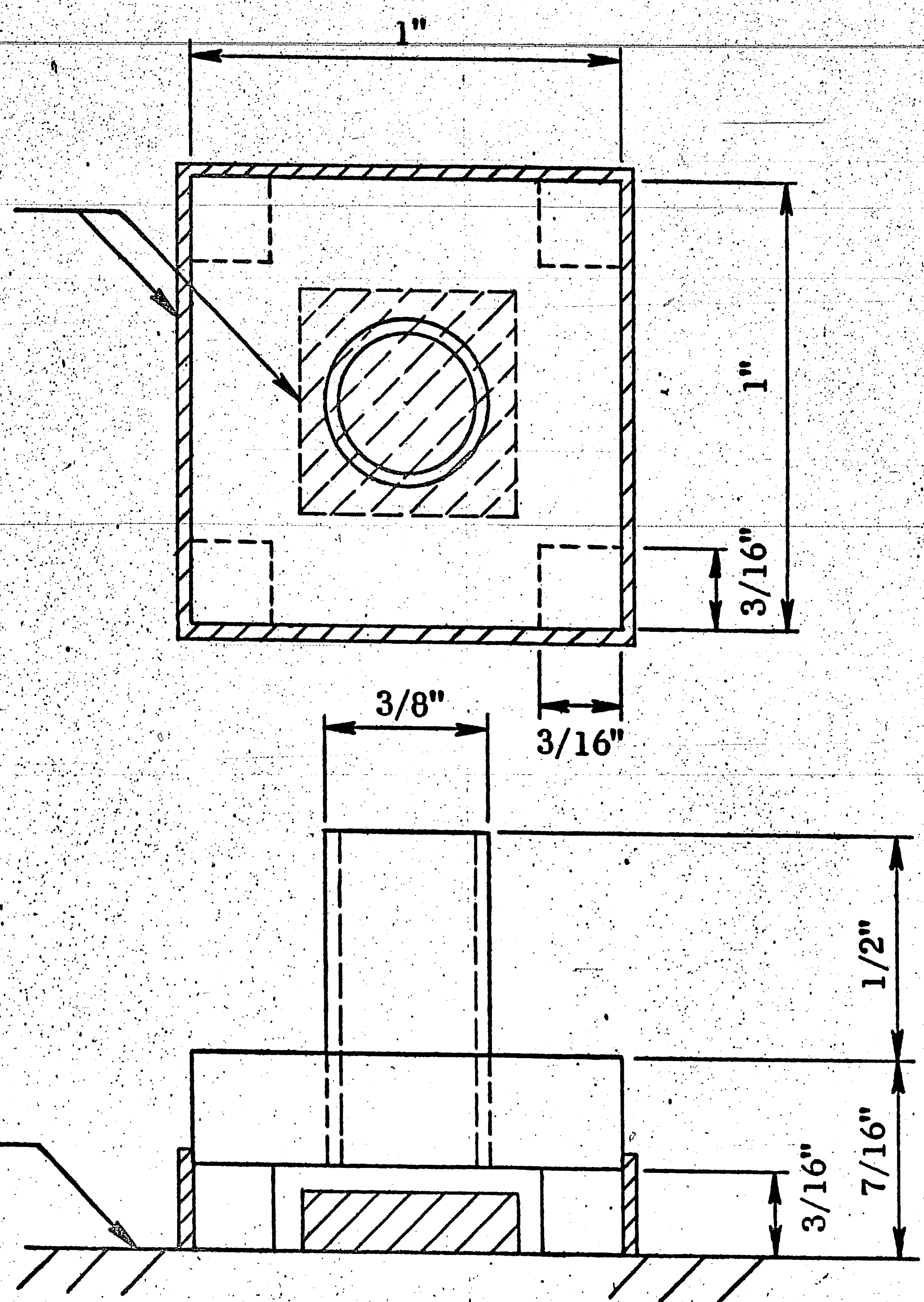
TIMING CIRCUIT

Fig. 5



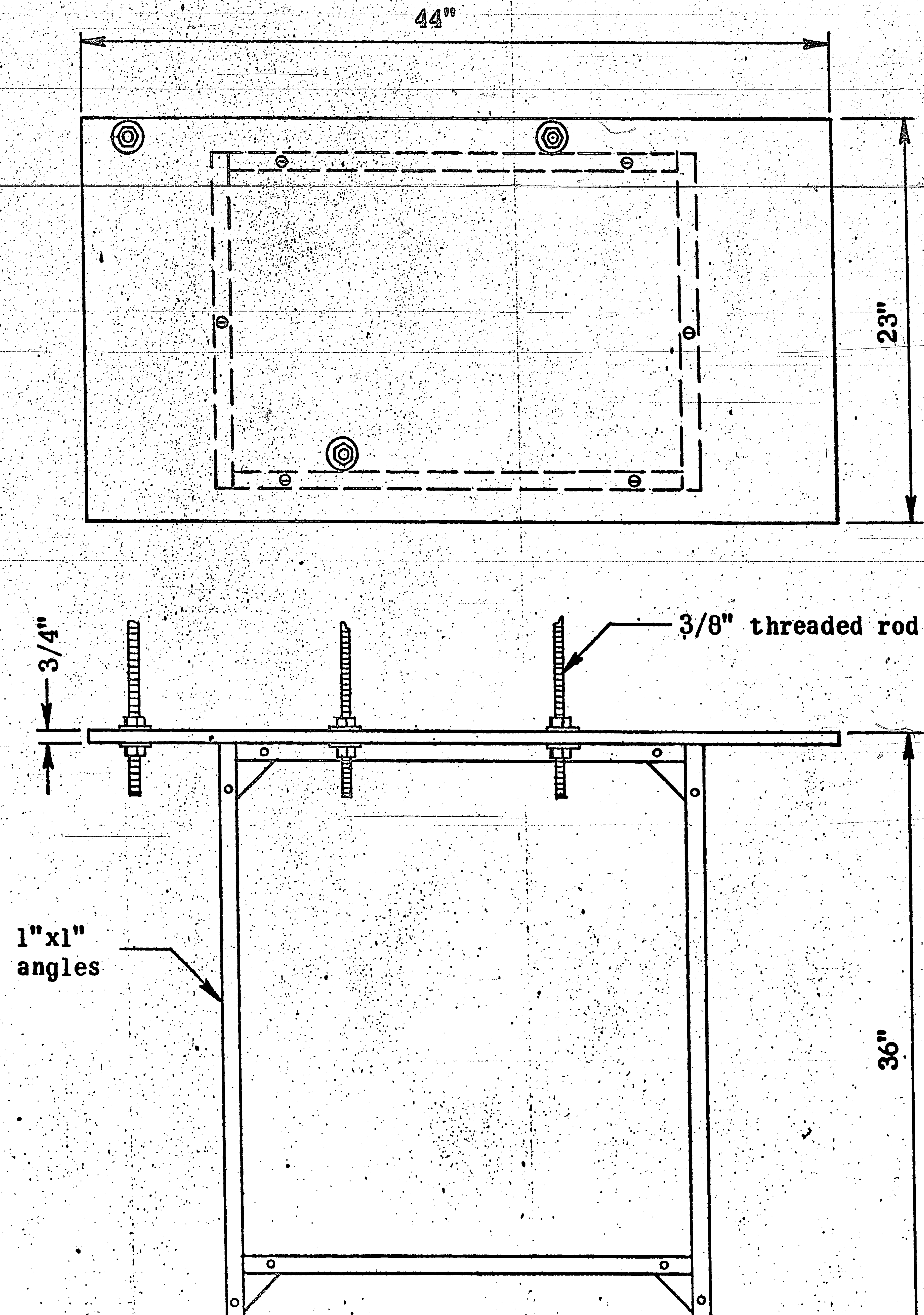
MAIN SUPPORT TABLE

Fig. 6



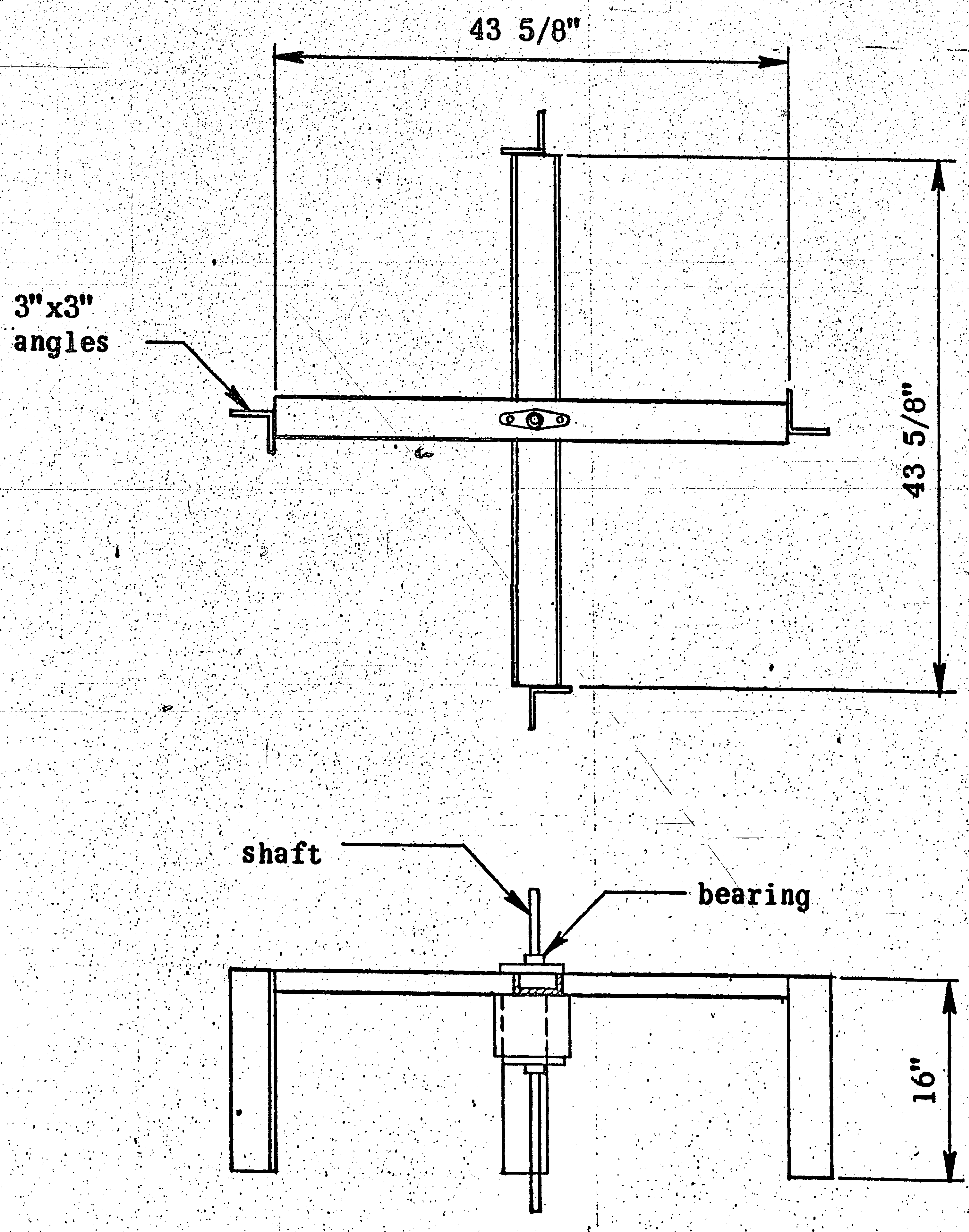
Discharge Box

Fig. 7



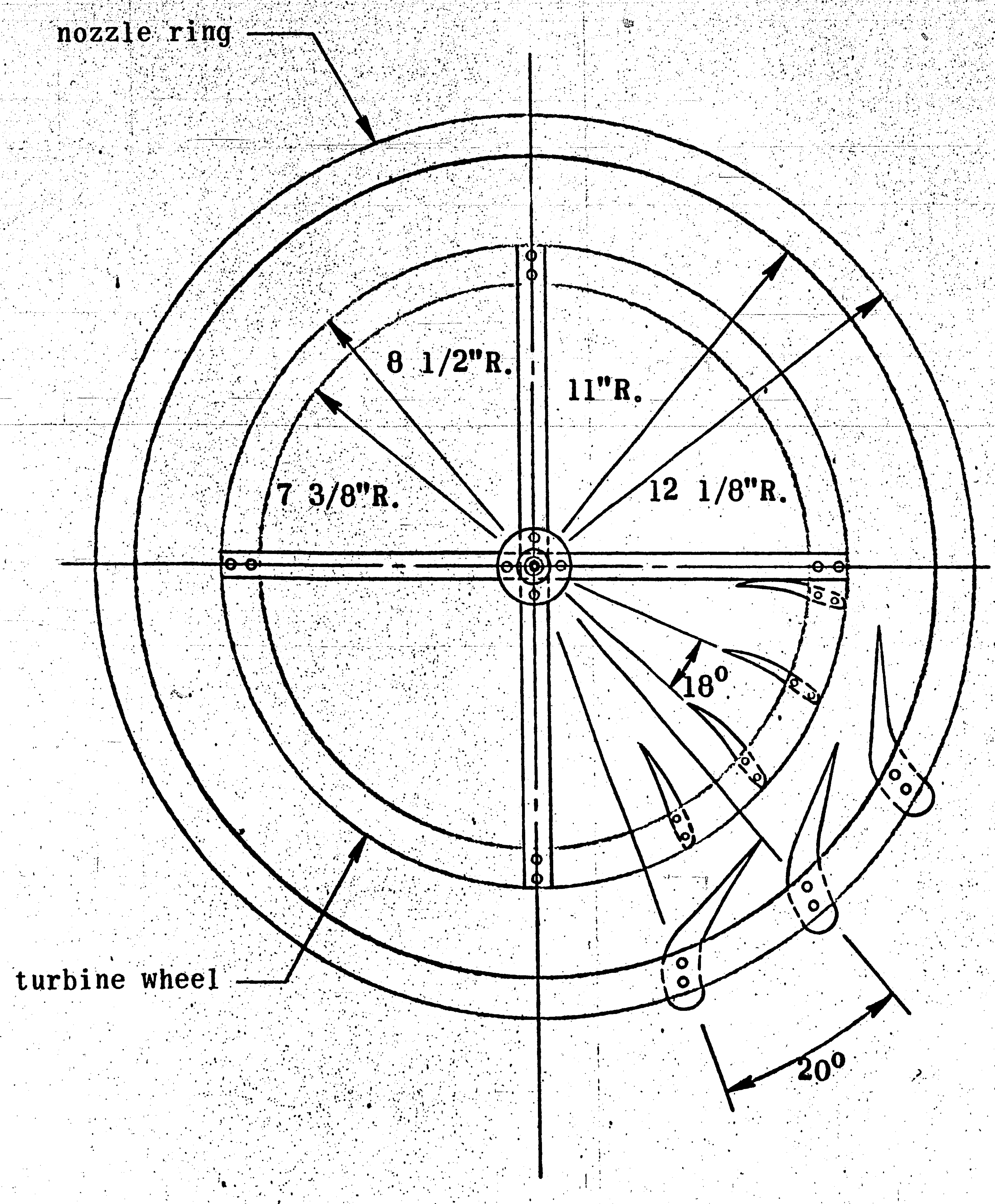
SUPPLY TANK TABLE

Fig. 8



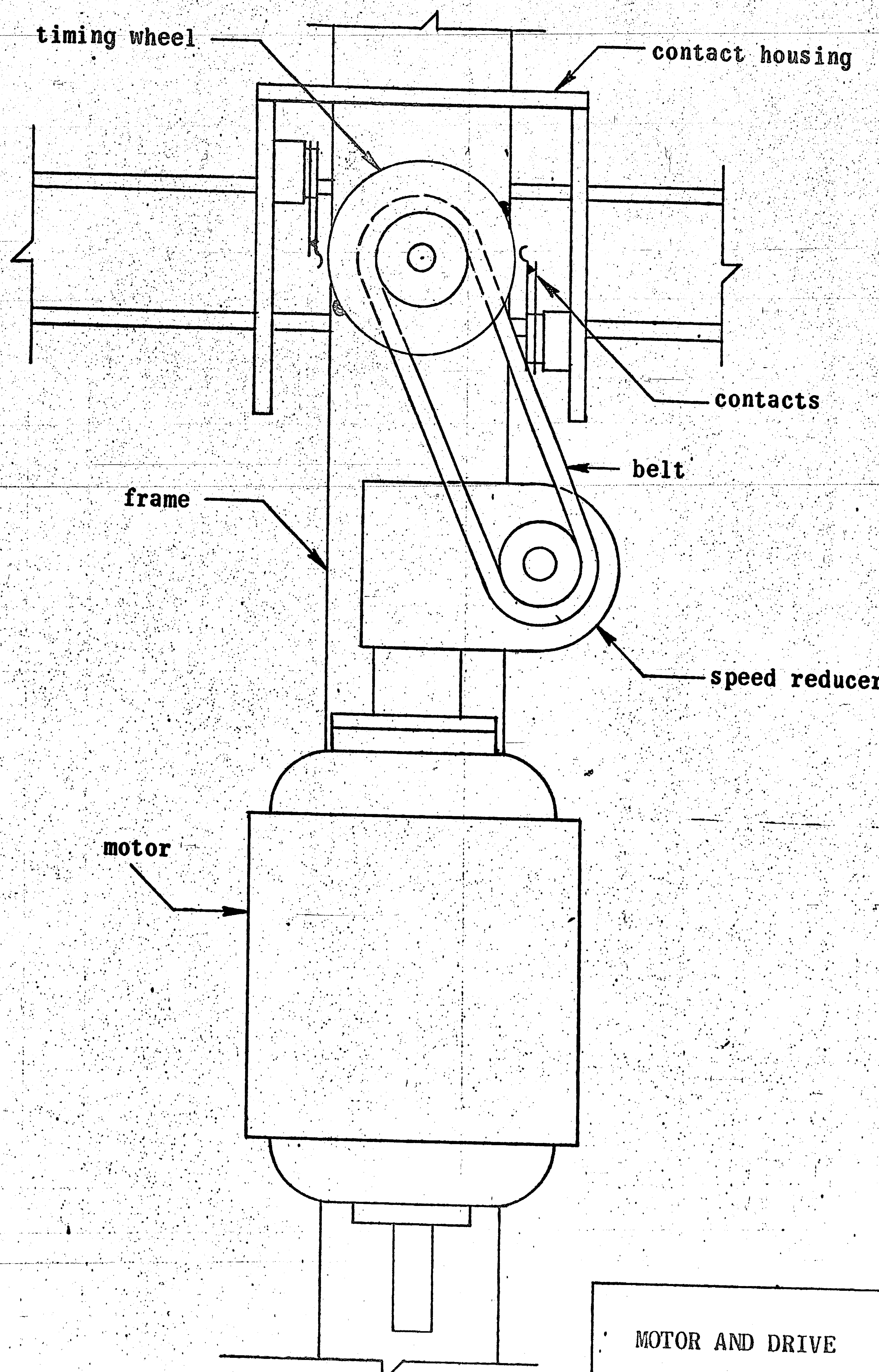
OVERHEAD FRAME

Fig. 9



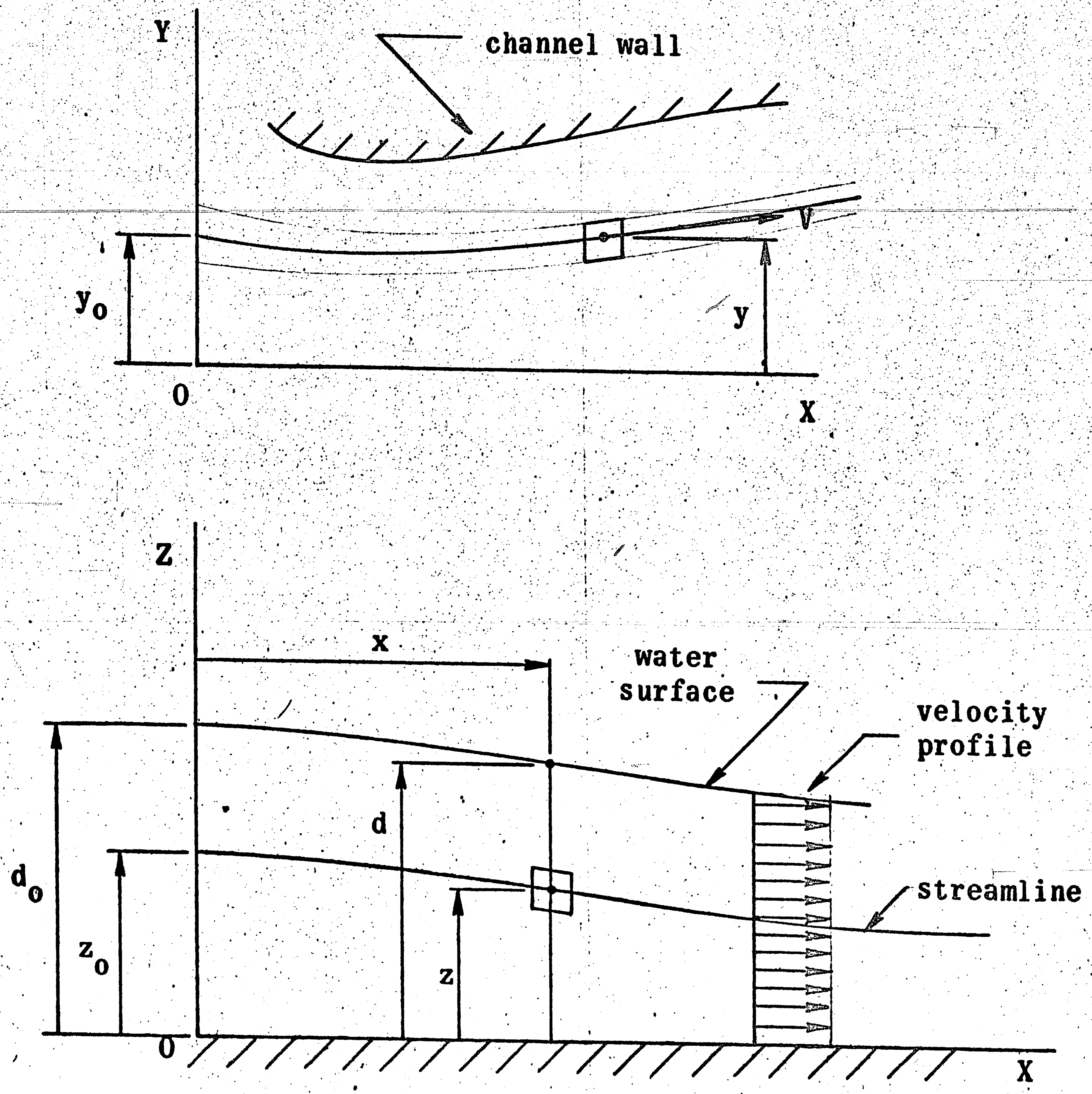
BLADE AND NOZZLE
ASSEMBLIES

Fig. 10



MOTOR AND DRIVE

Fig. 11



<u>Note</u>:

$x = 0$, y_0, z_0 is at an infinite reservior of zero velocity.

Figure 12

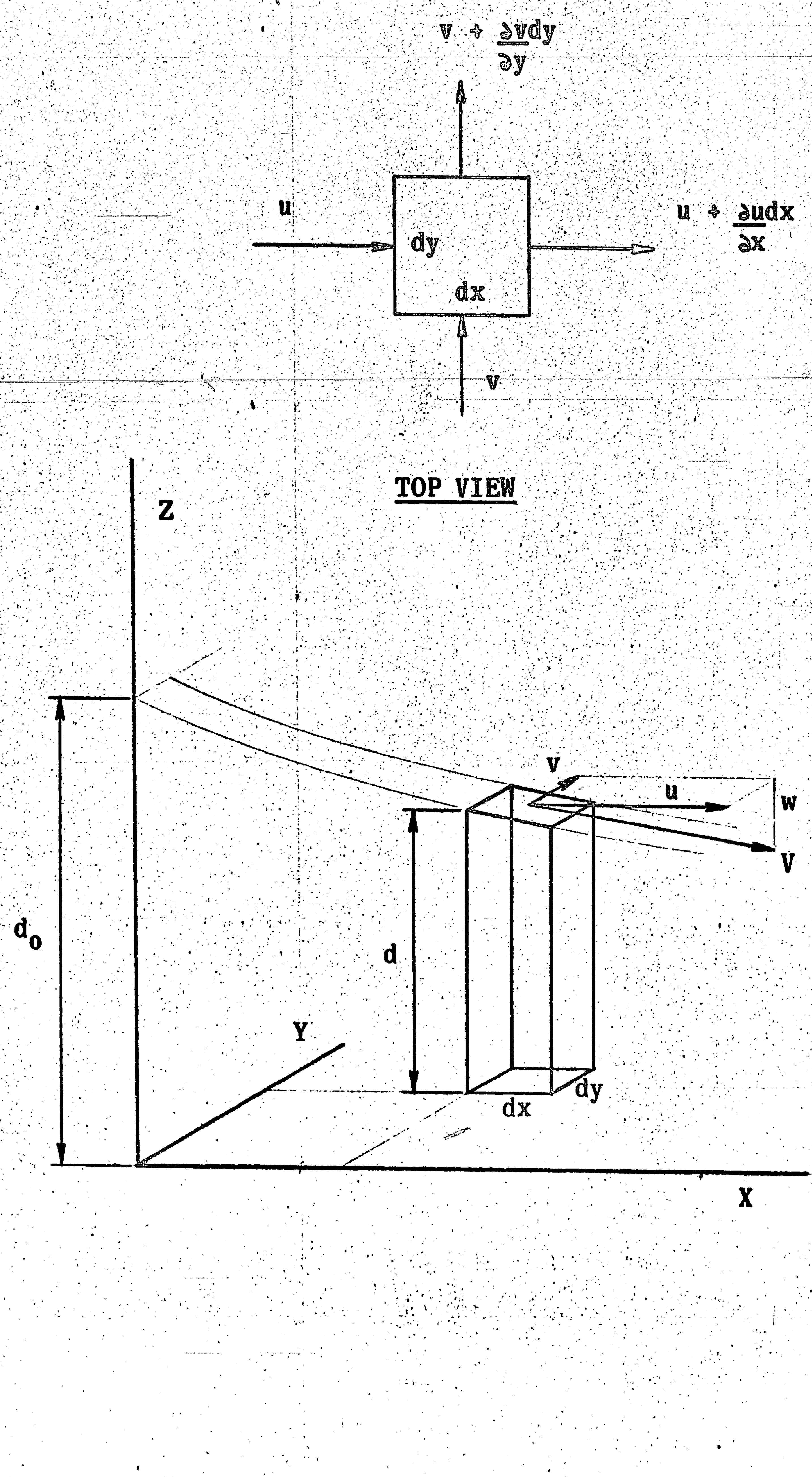
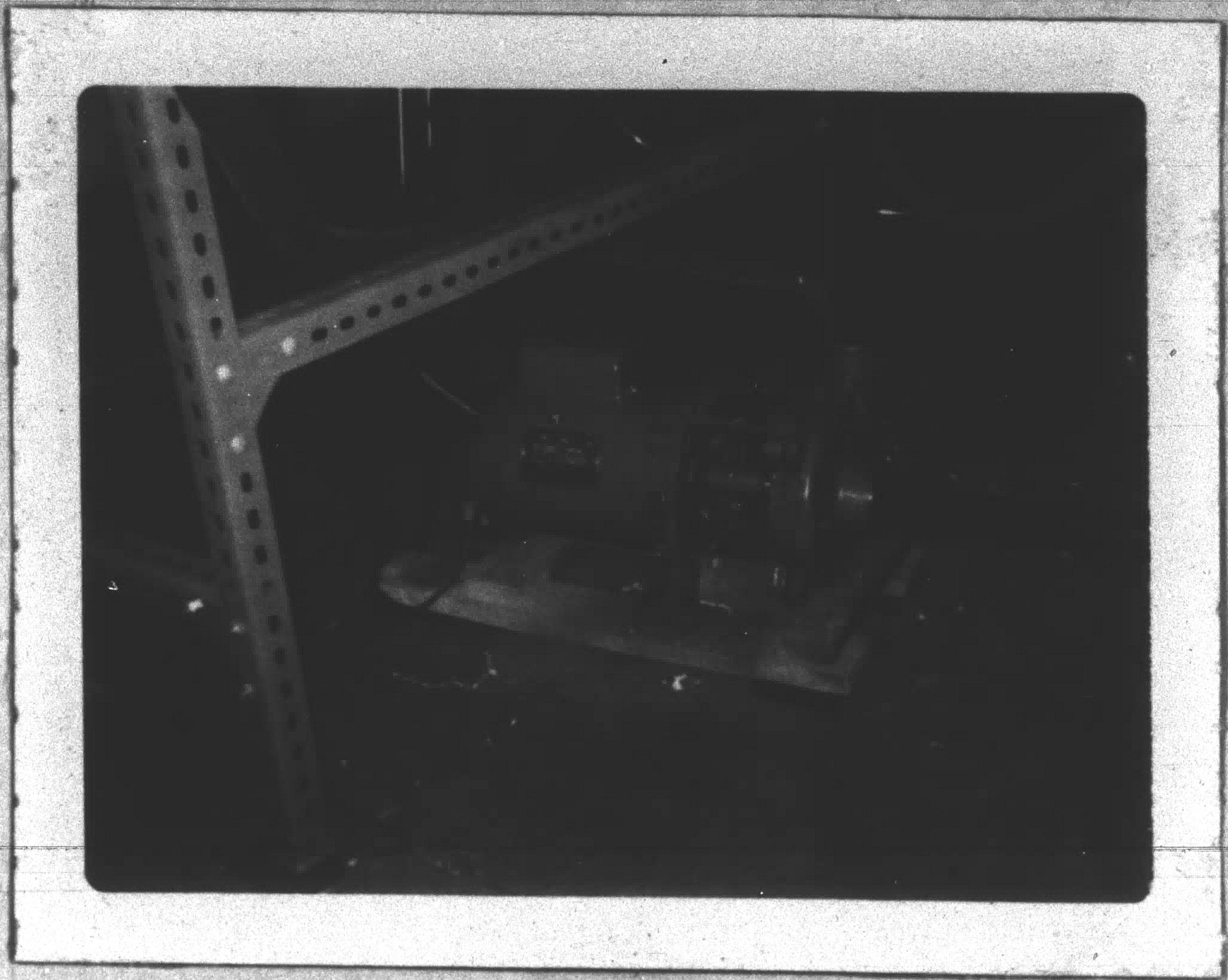


Figure 13



PUMP

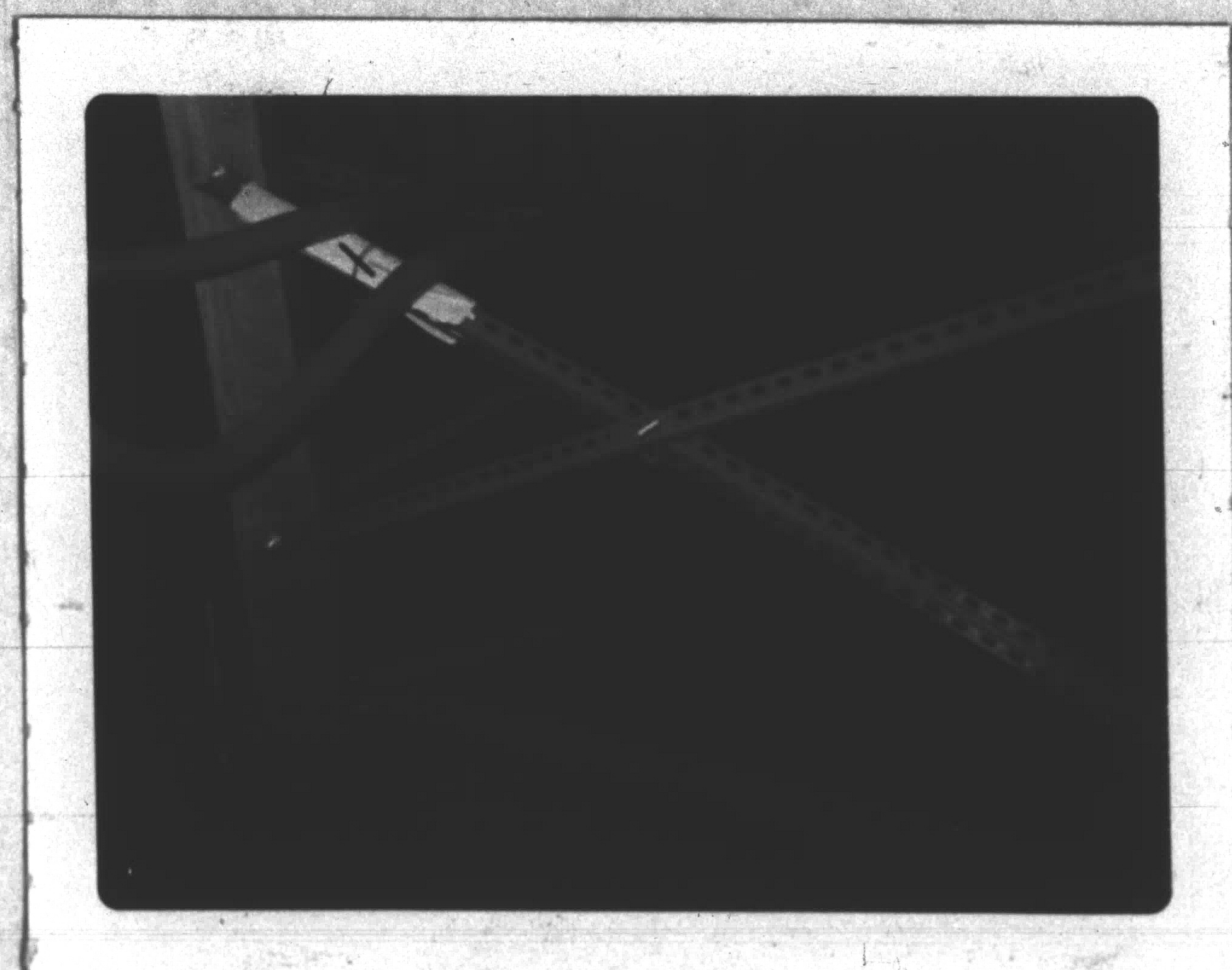
SPEED CONTROL AND CLOCK

BLADES AND NOZZLES

Fig. 14

PHOTOGRAPHS

MOTOR AND DRIVE



COLLECTING TANK

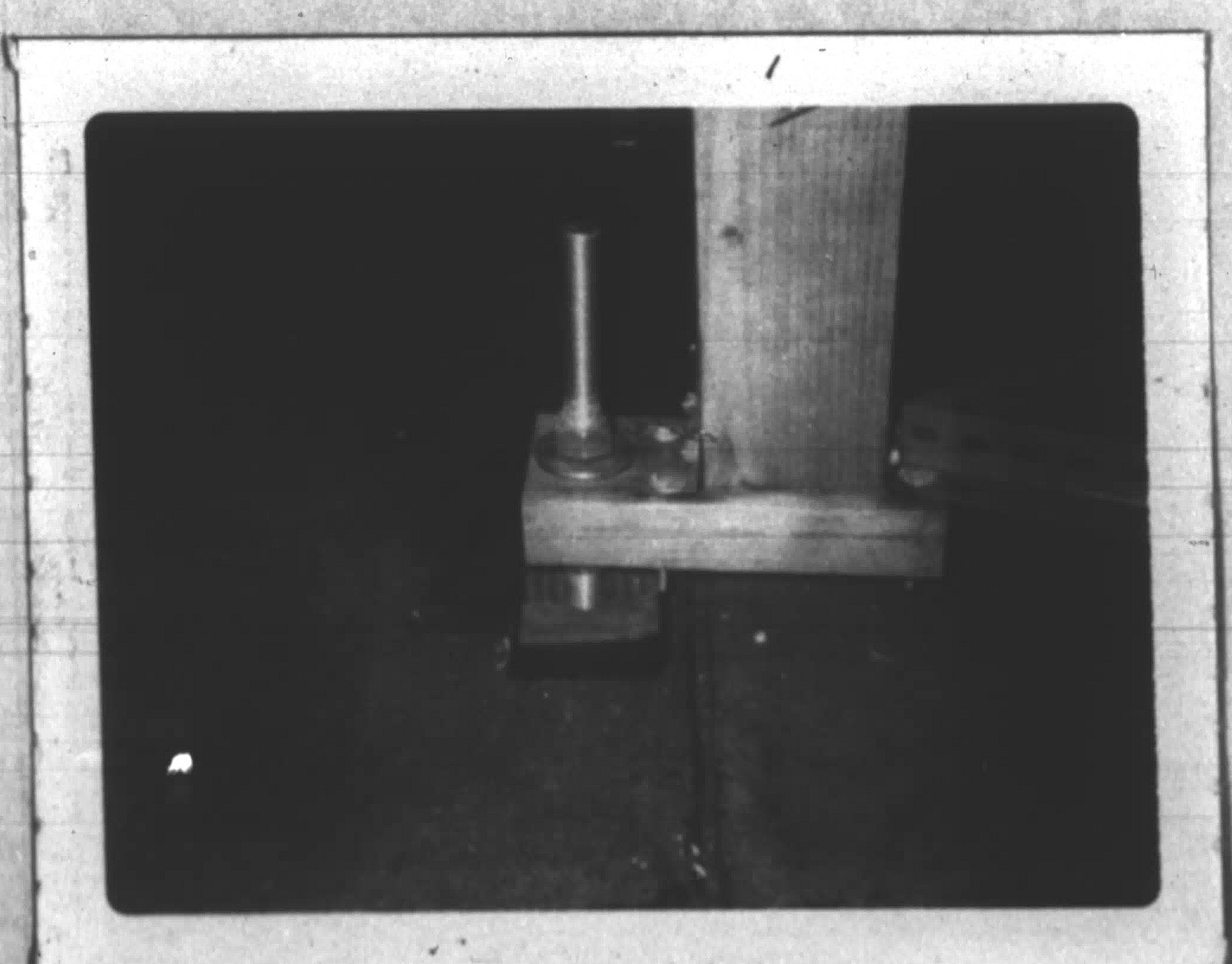
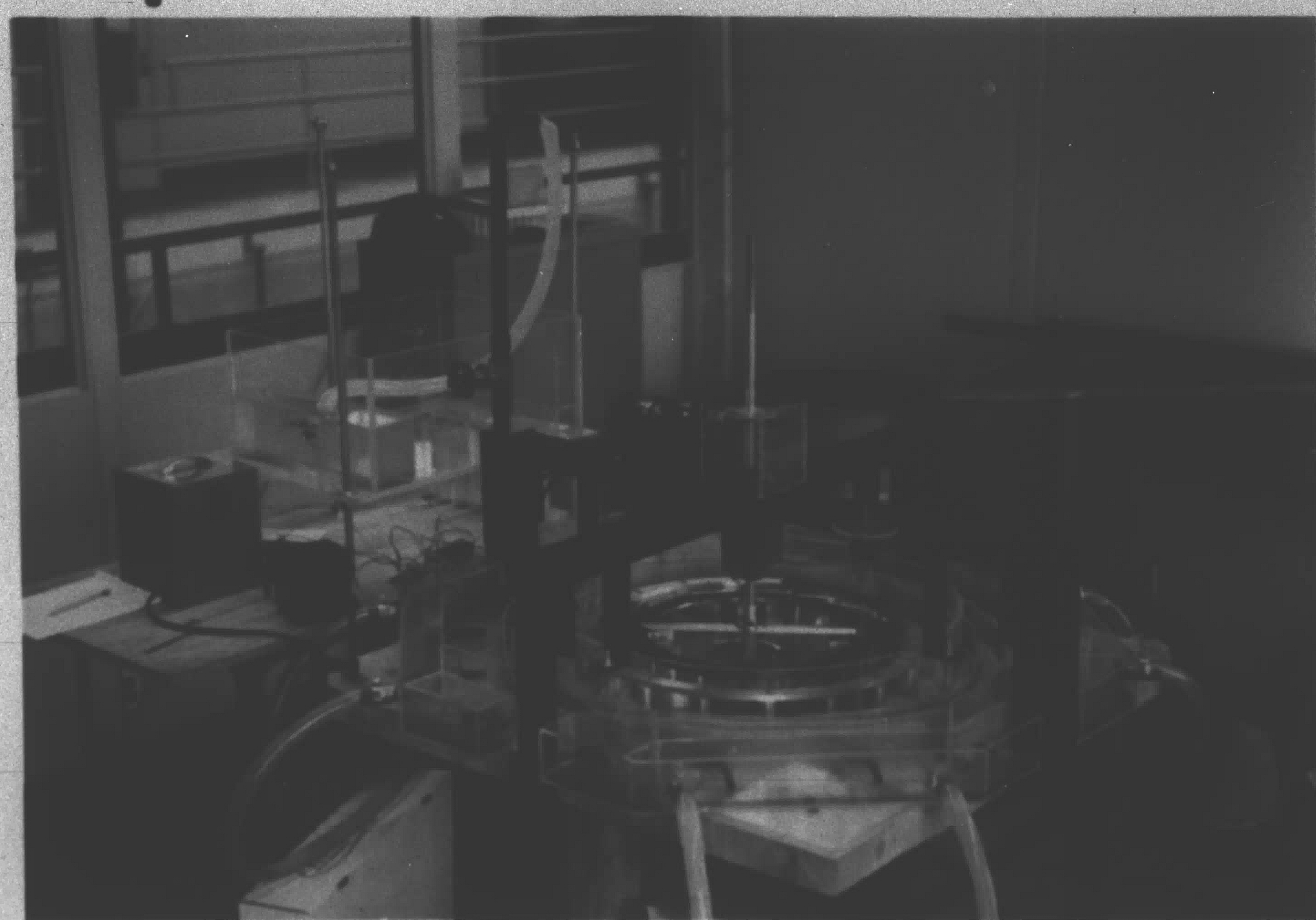


TABLE ADJUSTING SCREW

Fig. 14
PHOTOGRAPHS



ASSEMBLED APPARATUS

Fig. 14

PHOTOGRAPHS

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VITA

The writer was born on November 6, 1941 to Charles and Cathleen Ickes. He attended several grammar schools in New Jersey and Pennsylvania and Morristown High School in Morristown, New Jersey. In June 1963 he graduated from Newark College of Engineering with a Bachelor of Science degree in Mechanical Engineering. As an undergraduate he was a member of Tau Beta Pi and Pi Tau Sigma honorary fraternities and graduated Cum Laude. Since graduating he has served in the United States Army and has held jobs as an industrial engineer, production foreman, and mechanical engineer.